Similarly,

$$x_{n-1}=\frac{xQ_{n-2}-P_{n-2}}{P_{n-1}-xQ_{n-1}},\ x_{n-2}=\frac{xQ_{n-3}-P_{n-3}}{P_{n-2}-xQ_{n-2}},\ldots\ x_{n-k}=\frac{xQ_{n-(k+1)}-P_{n-(k+1)}}{P_{n-k}-xQ_{n-k}}.$$

$$\therefore\ x_nx_{n-1}\ldots x_{n-k}=\left[\frac{xQ_{n-1}-P_{n-1}}{P_n-xQ_n}\right].\left[\frac{xQ_{n-2}-P_{n-2}}{P_{n-1}-xQ_{n-1}}\right].\left[\frac{xQ_{n-3}-P_{n-3}}{P_{n-2}-xQ_{n-2}}\right].$$

$$\left[\frac{xQ_{n-4}-P_{n-4}}{P_{n-3}-xQ_{n-3}}\right]\ldots\ldots\left[\frac{xQ_{n-(k+1)}-P_{n-(k+1)}}{P_{n+k}-xQ_{n-k}}\right]$$

$$=(-1)^{k+1}\left[\frac{P_{n-(k+1)}-xQ_{n-(k+1)}}{P_n-xQ_n}\right]=(-1)^{k+1}(A),\text{ suppose.}$$

$\therefore\ (-1)^{k+1}x_n\times x_{n-1}\ldots x_{n-k}=(-1)^{2k+2}(A)=A=$result stated.

Also solved in the same manner by *G. W. GREENWOOD.*

GEOMETRY.

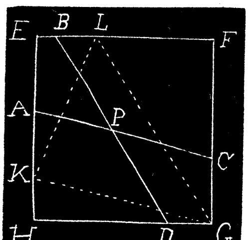
195. Proposed by F. L. SAWYER, Mitchell, Ontario, Canada.

The diagonals of a four-sided figure are h and k, and the area is A; show that the area of the circumscribing square is

$$\frac{h^2k^2-4A^2}{h^2+k^2-A}.$$

Solution by J. R. HITT, Principal Liberty High School, Goss, Miss.; J. SCHEFFER, A. M., Hagerstown, Md.; and L. L. LOCKE, Professor of Mathematics, Adelphos College, Brooklyn, N. Y.

Let $AC=h$, $BD=k$, be the diagonals of the four-sided figure, and $EFGH$ the circumscribing square. Draw GK, GL, parallel to h, k, respectively, and denote HK, FL, by a, b, respectively.



$\triangle\ GKL=\frac{1}{2}GK.GL\sin KGL=\frac{1}{2}hk\sin P=A$. If $x=$ side of square, we have, $\triangle\ GKL=A=x^2+\frac{1}{2}ax-\frac{1}{2}bx-\frac{1}{2}(a+x)(x-b)=\frac{1}{2}(x^2+ab)$.

Hence, $b=1/a(2A-x^2)$.....(1).

Also, $GL^2=k^2=x^2+b^2$, $GK^2=h^2=x^2+a^2$.

Hence, $a^2=h^2-x^2$.....(2), and, by adding, we get $2x^2=h^2+k^2-(a^2+b^2)$.....(3). Substituting in (3) the values of b and a from (1), (2), we have, $2x^2=(h^2+k^2)-(2A-x^2)^2/(h^2-x^2)-(h^2-x^2)$, whence, $x^2(h^2+k^2-4A)=h^2k^2-4A^2$.

Therefore, $x^2=\dfrac{h^2k^2-4A^2}{h^2+k^2-4A}$

Also solved by *G. B. M. ZERR.*